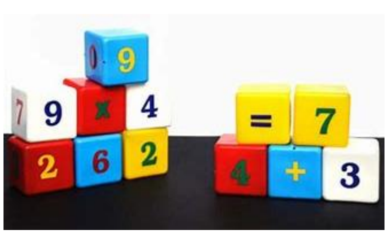
PoCo

A Polyhedral Computing Experience

The space is unfolding in front of your eyes: a 3D simulation that will change the way you observe the real world. You manipulate 3D complex structures with unseen versatility. You experience pure communications with the space itself. You see a triangle, it is a triangle. No confusions, no gimmicks.

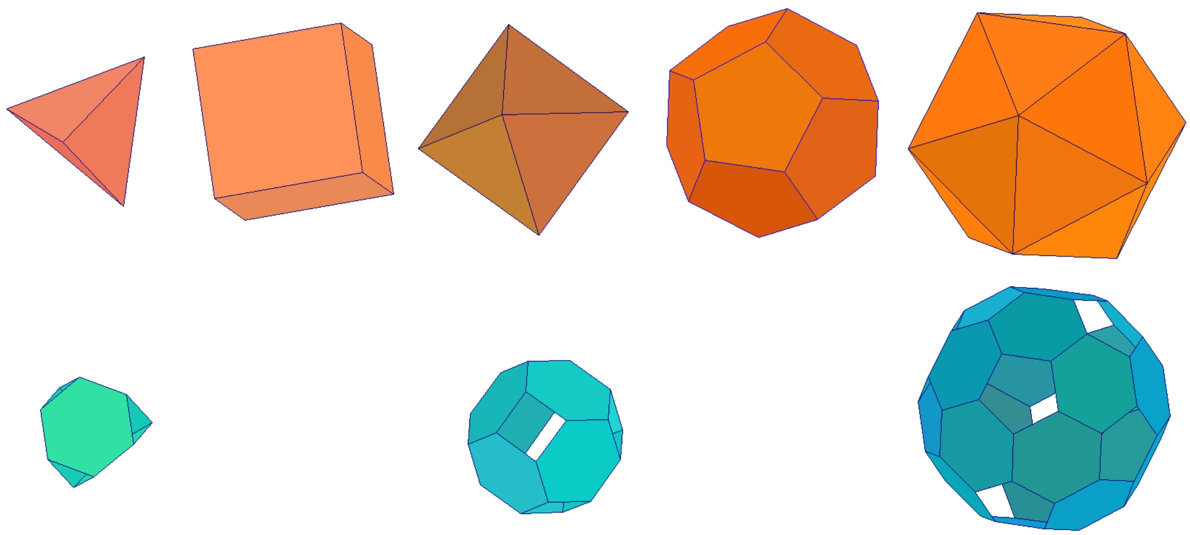
I. Hello PoCo!



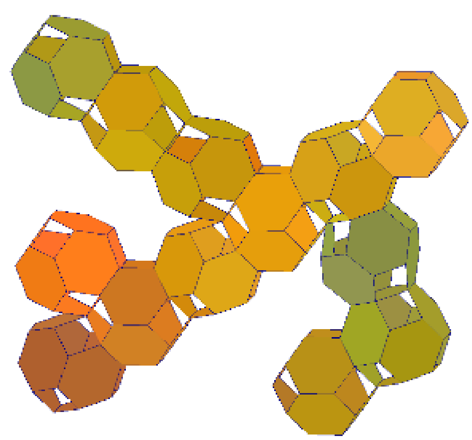
PoCo is a computer program that allows you to play with polyhedra. Do you remember playing with cubes when you were a kid?

2. PoCo Playground

Now you can play with many more complex objects: polyhedra and ensembles of polyhedra. Here you have the basic constituents of our playground.



On the first row, notice the Platonic solids. Implied by the number of faces (4,6,8,12,20), their names are tetrahedron, cube (or hexahedron), octahedron, dodecahedron and icosahedron. On the second row you see truncated versions of three of the Platonic solids (matching to the ones on the top row). They are Archimedean solids, obtained by truncation, or cutting corners, of the corresponding Platonic solids. They have in common something special: all faces are regular hexagons, of the same size. That will allow future interactions and the generation of large colonies of polyhedra. Once you have a few operations available, you can create “geometric expressions”, by applying sequentially many operations. Such a geometric expression is illustrated on the right.

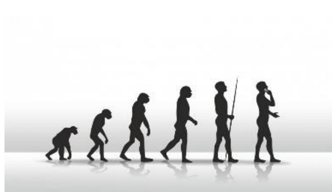


Bees are wonderful mathematicians. They realized that the most efficient structure to be used for constructing rooms with least material is the hexagonal structure. Humans were able to scientifically acknowledge that just recently.

Operations are the key components to evolution from elementary objects to colonies. Working together as a group, ants, bees and humans are the best examples.

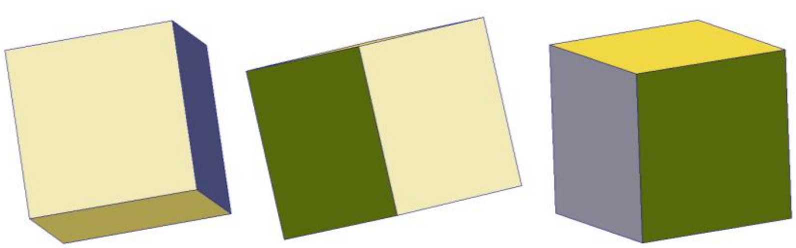







PoCo allows you to perform operations and transformations on geometric objects. To better understand the structure of the complex colonies of geometric objects, PoCo can show you the third dimension. You can move, rotate and transform objects to understand their geometry.

For instance, a cube can have several snapshot images:

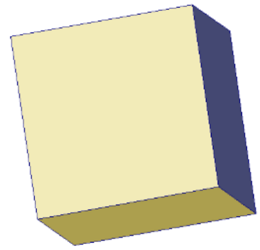


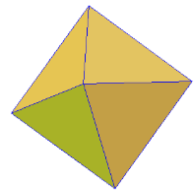
The cube is rotating around its vertical axis, to the right. Follow the yellow face for instance. It is fully visible in the first image, and it becomes smaller in the second and even vanish in the last one.

3. Simple Transformations

So, what kind of transformations/operations can you perform on geometric objects? Can you apply these operations recursively?

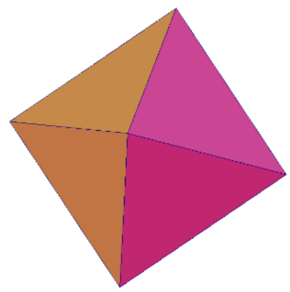
- **Dual Transformation**
  The dual of a convex polyhedron is obtained by taking the centers of its faces and connecting them if the corresponding faces of the original are adjacent.

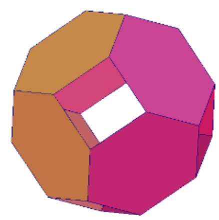


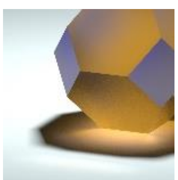


Duality is represented in art, too. Sometimes it refers to similarity and sometimes to opposite concepts. For instance, in the Star War world, the duality is represented like this image. The force and its dark side are parts of the same face.

- **Truncation**
  Removing parts of the objects via planar cuts will create more complex shapes: more vertices, more edges and more faces.

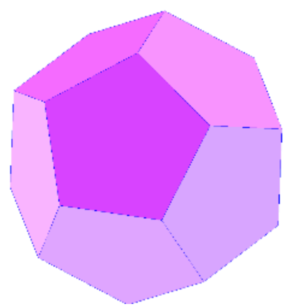


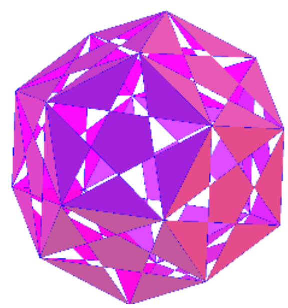


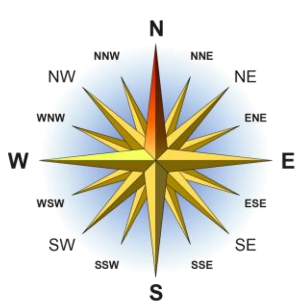


Truncation or cutting appear in our daily life. It can be about a cake, about an image or about a 3D shape.

- **Star Faces Transformation**
  Each face of a polyhedron is a polygon. Changing the faces into star polygons using vertex jump at various distances creates an artistic view.

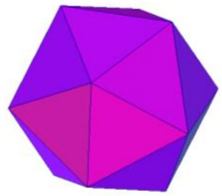
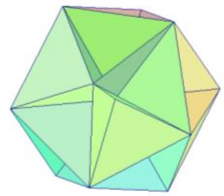
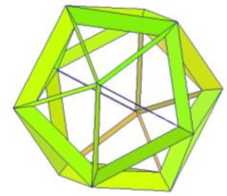


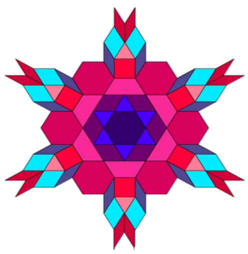




Star polygons occurs naturally in the design of the Compass Rose.

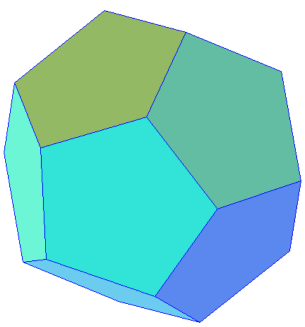
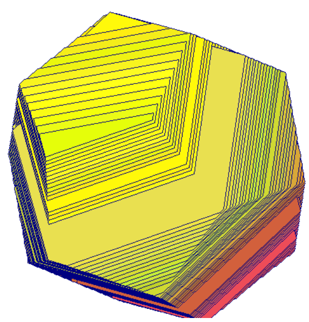
- **Star Transformation**
  The star of a polyhedron is obtained by creating a new face for each of its edges by connecting it to the polyhedron center. The connection can be fully or partially realized.




Polygon star transformations are commonly encountered in decorative art.

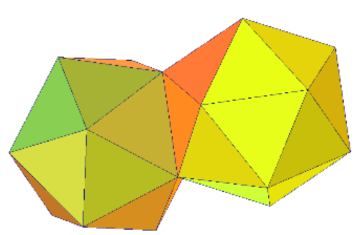
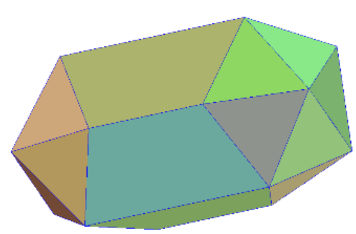
- **Slice Transformation**
  An apple can be sliced into layers. These layers are geodetic (same height) views of the apple. Similarly, any geometric shape has a layered representation.

Take a dodecahedron and a 100 layers view of it.

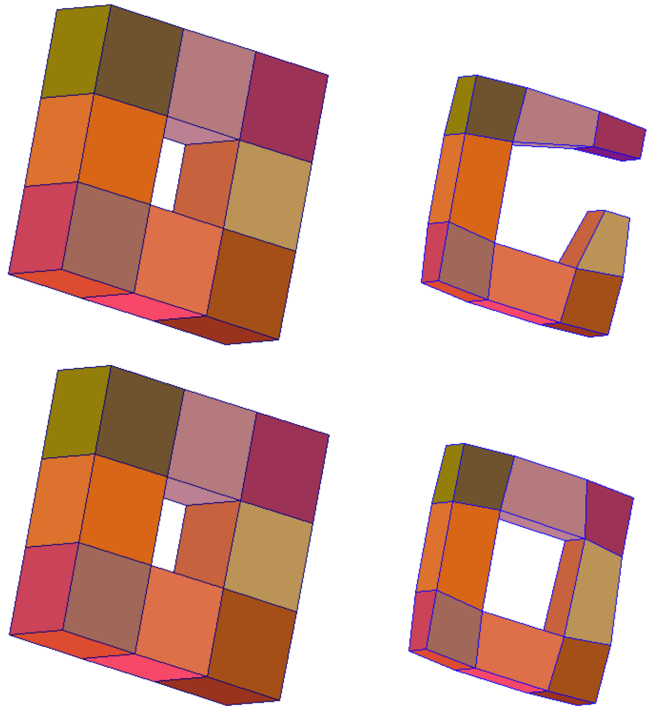
- **Convex Hull and Compact Transformations**
  The most illustrative example of the convex hull is “gift wrapping”. That refers to the smallest box that is a convex polyhedron and contains gift.

For polyhedra, this concept translates to an algorithm that associate to any given 3D object its convex hull.

- **Weld & Simplify Transformations**

The images on the right display two dual transformation of ensembles consisting of 8 cubes. While the first one has a chain structure, the second one is structured as a ring. This information is not visible for the original objects, but their dual clearly indicate this difference. The idea is that information that is not detected some structures can be easily spotted in their dual representations. Simplification typically refers to removing redundant information. For instance, if a face is recorded twice, the geometry is not different. So, simplification will remove the duplicated faces.

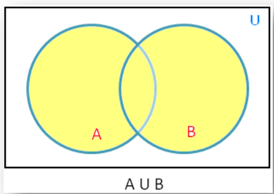


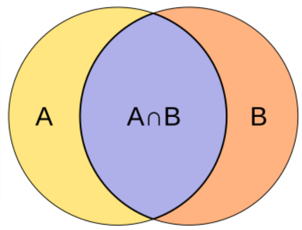
Welding is one of the most common industrial operation of joining objects that are made of metal. It appears in the construction of transcontinental oil pipelines, large bulk carriers, sky scrapers and cars.

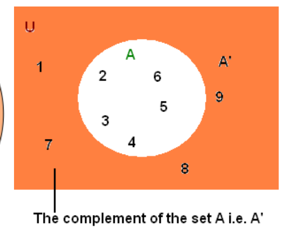
4. Operations

Operations are performed for collections of elements of size 0, 1,2 or more. Examples of operations could come from arithmetic and algebra: 2+3 and 5*6. A sequence of operations can be applied to construct expressions like 2+3+5*6. Some theories consider operations with no arguments: the constant number are falling into this category. Changing the sign of a number can be viewed as an operation with one argument. Addition and multiplication are examples of operations with two arguments, and we can go beyond that, for sure.

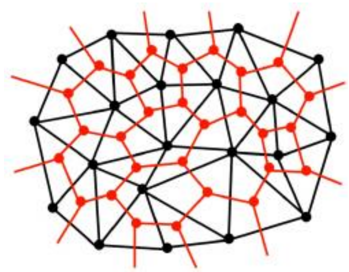
Set theory is based on a different kind of operations. Union, intersection and taking the complement are the most common ones.

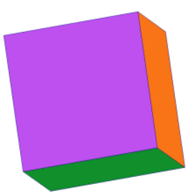


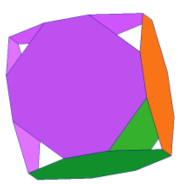




Graph theory is another source of good examples for operations: the product of two graphs, the construction of the planar dual. Notice the blue graph on the right. For each region create a red dot in the center. Connect two red dots with a red line if they correspond to adjacent regions. The red graph is the dual of the blue graph. You should visualize that the edges emerging to the outside region are connected to one new dot.

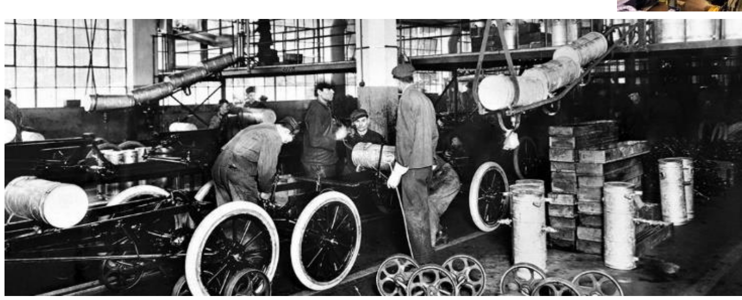






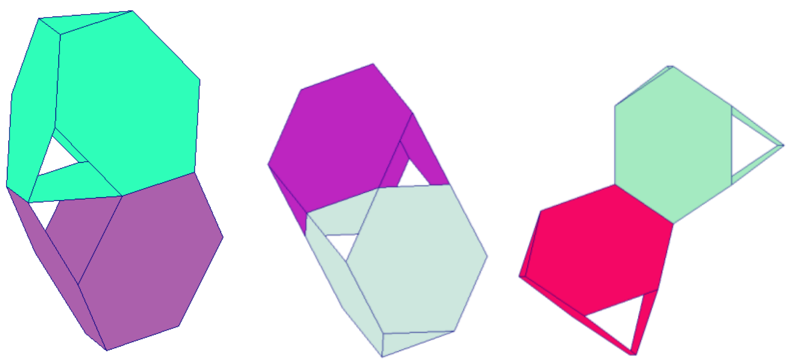
Geometry allows us to apply operations to two- and tri-dimensional shapes. On the left, you may observe a cube and its truncation.

Another example of operations could be found at the assembly line in car factories. Operations are applied in sequences and typically start from a base (sassy) and add components one by one: the engine, the doors, the wheels, the windshield. The first time the assembly line was used goes back to Ford car maker in 1913.



We introduce a few operations inspired by the examples illustrated above for the set of elements introduced in Section 2 - "PoCo Playground". These operations consist of connecting elements and this can be done in several ways. Consider that the elements have facets (full body faces) and cofacets (empty shapes that emerge while truncating the original solid). Connections could be done by joining two facets or two cofacets. It is possible to connect two elements of different kinds, if they have similar facets. As cofacets are specific to only one kind of elements they can be merged only for elements of the same kind.

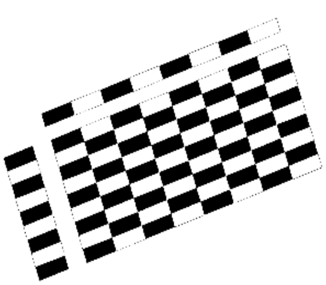
It is also important to notice that connecting two facets can be done in two ways, depending on the type of adjacent created for the faces of the new object. If by connecting the faces their neighbors will match, we call the operation of type ***Delta***, and otherwise we call it ***Gamma***. Operations that connect cofacets are of type ***Nabla***. The images below illustrate these three operations:



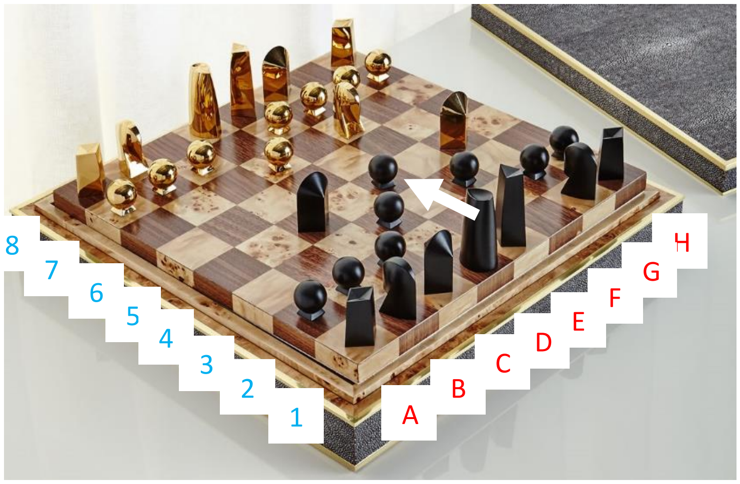
5. Cartesian Products

Obvioulsy, the multiplication table is a good example of Cartesian products. Notice the color change: from red=small to green=large. Can you identify the pattern here?

X	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

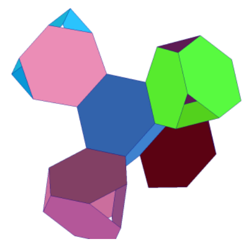


This chessboard is another example of a Cartesian product. It is the product of one row and one column, each of them having 8 square cells. The result is a chessboard with 64 cells. A special system aimed to designate the cells of this chessboard is used by chess players record their games, so that they will remember all the moves to the extend they can replay the entire game. If we designate the raw with 1-8 and the columns with A-H, the chess players will be able to this. E2-E4 simply means moving the white pawn of the king side two squares head.

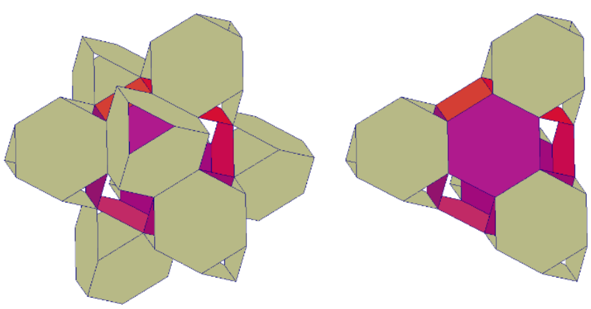


From a different perspective, we can visualize that starting with a column of 8 cells and replacing each of its cells with a row, which has also 8 cells, and then the board is reconstructed. But, so far, we have not taken into consideration the color of the cells. This is additional information and can be added as a "painting" operation: paint all cells in white or black such that no two adjacent cells have the same color.

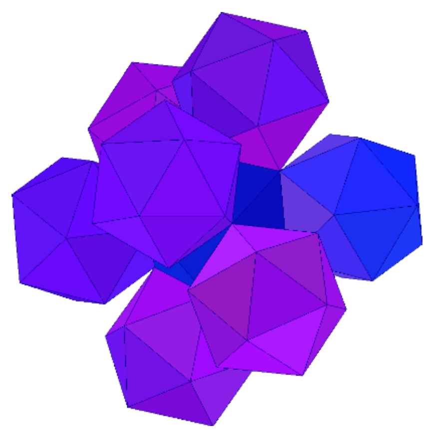
Similar to the rows and columns of the chess board, our elements have cells: facets and cofacets. We construct the product between two objects by replacing each of its cells with objects of the second kind. For instance, starting from a TTC module and plaquing it with TTC modules using Nabla operations we obtain the PowerNabla(TTC,TTC) which is depicted on the right. Follow the **User Notes** of the software package for a complete description of the Geometric Cartesian Products.



Partial Products. You will notice that some combinations are not be realizable in the tri-dimensional space. Partial products are obtained by plaquing objects on a subset of their faces. A 4/8 product of an O module with T modules is illustrated below.



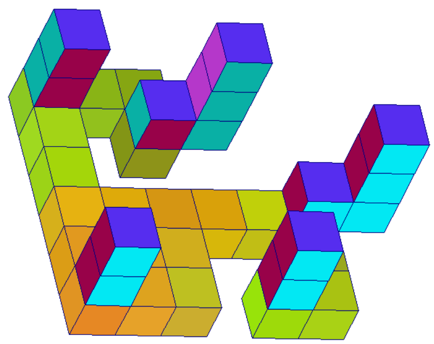
An 8/20 product between an I module with itself is shown here:



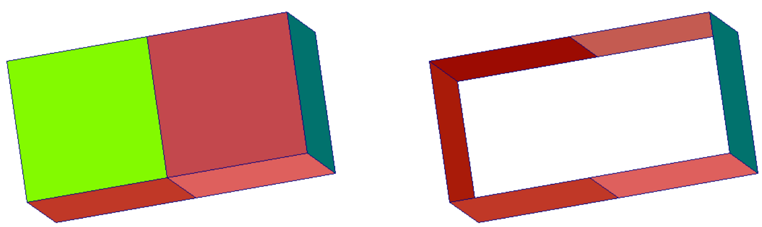
See the PoCo documentation (under Help/Manual menu), Appendix D, for a complete list of Cartesian products with PoCo polyhedra.

6. Builders

Builders allow you to create complex configurations fast. You can make copies of an object as you move it, or you can ask for help from your assistant builder. The assistant will interpret your actions considering the current status and will find the best choice to continue. You can override its suggestion it that is not what it is expected. If needed, you can revert your last move. Builders use as default the construction module that is selected, so before you start using a builder configure the default building module properly. You can select it from the available collection or from the ones already generated by you.

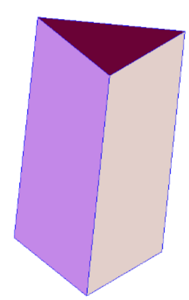


Deleting vertices, facets or cofacets is also be part of the building process. For instance, you can generate a frame by joining two cubes and deleting faces (2 in front, 2 in the back and 1 in the middle).



7. Puzzles

The construction of many configurations is intuitive, and it is easy to figure out the sequence of operations that was used. However, there are simple configurations that are difficult to reverse engineer. How would you generate a triangular prism?



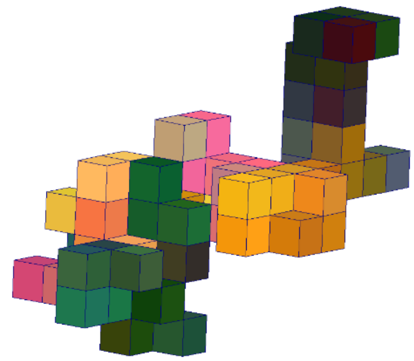
Your answer might be:

- generate a delta operation for two O modules,
- delete all lateral faces to obtain two triangles that are parallel
- apply the convex hull and compact the faces

Periodically, propose such puzzles as you experiment with **PoCo** and learn new things. Share them with your friends.

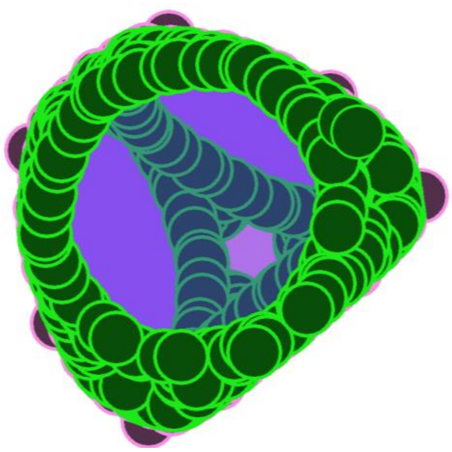
8. Evolution

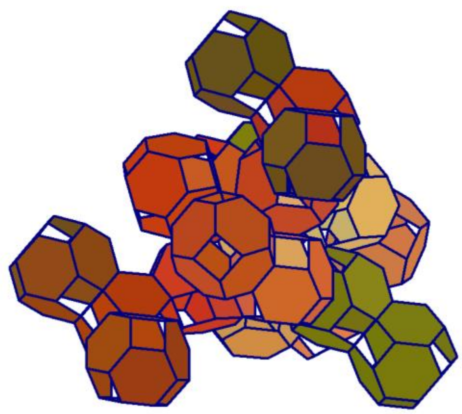
Probably the most intriguing feature of PoCo is its ability to create large scale simulations. Say that you want to play with 100 cubes and ask PoCo to combine them using Delta operations to create on final complex module. One answer could be the following:

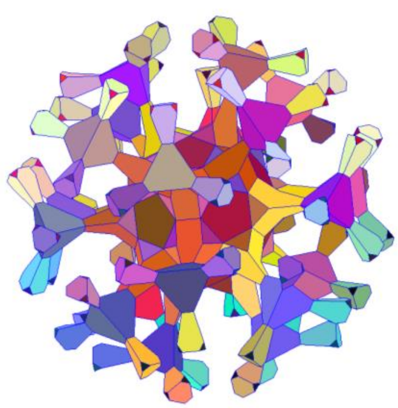


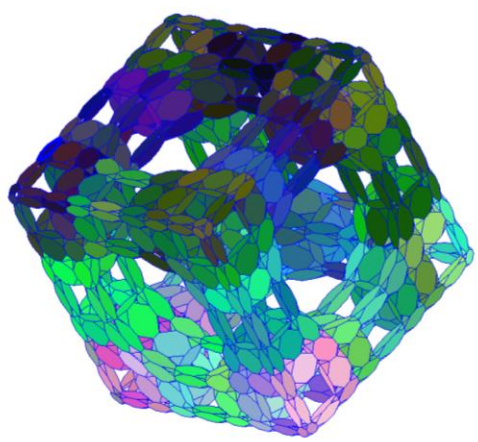
9. Project Diversity

Here, we include a series of projects that could be obtained using PoCo. Structures encountered in biology, chemistry and architecture could be a good source of inspiration.









Use your creativity to expand the list of interesting projects.