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Polyhedral Computing
A New Framework for Teaching Spatial Abilities in Young Students

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www.x-colony.com/xcolony

www.x-colony.com/PoCo

Abstract

Polyhedral Computing is a platform that allows students and teachers to manipulate geometric solids with various level of complexity. The goals of these activities are: improving spatial cognition and train for spatial intelligence. A project starts from a collection of Platonic and Archimedean solids and applies transformations and connecting operations to create complex ensembles of polyhedra. The system was implemented both physically (www.x-colony.com/xcolony) and virtually (www.x-colony.com/PoCo).

Spatial education programs, based on this platform, engaging fifth and sixth graders, showed that participants improved their spatial abilities and mathematical understanding. The scores improved by 26% for spatial reasoning, 15% for solution finding and 7% for comprehension. The increase in math scores in the next year of the students (now sixth graders) was 12% in standard evaluation tests.

Joint work with Cristian Voica and Consuela Voica.

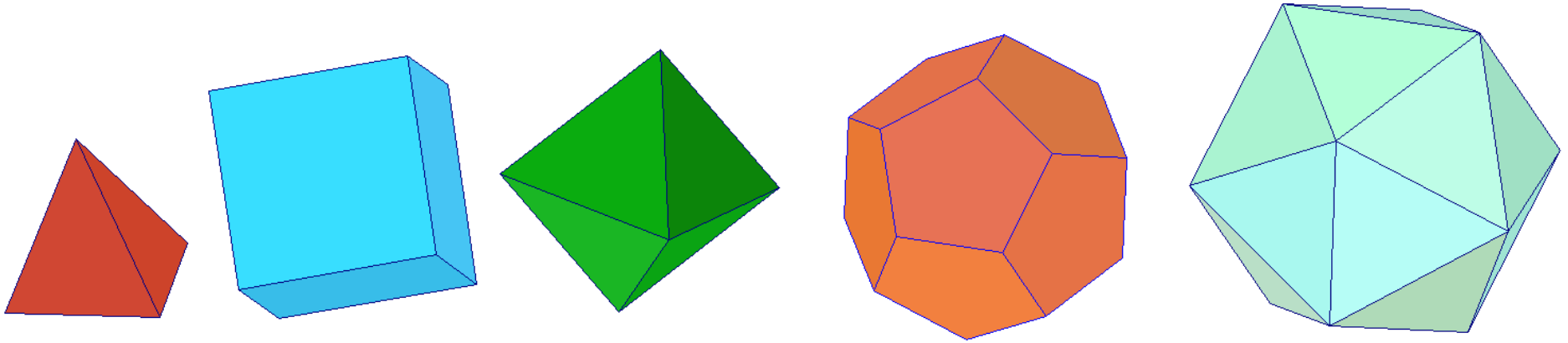
Outlines

HexPack and **PoCo** free licenses are available for the CRM9 partitipants:
http://x-colony.com/PoCo/index.php/resources/downloads/

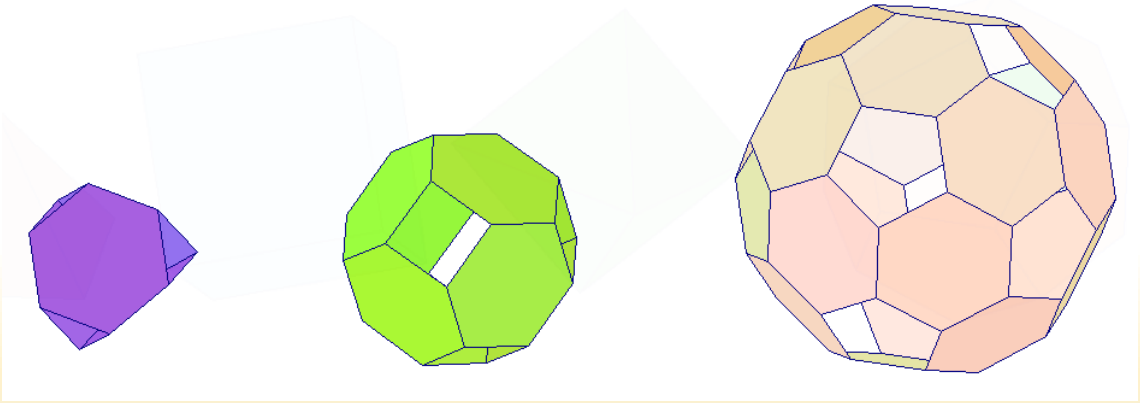
Download this presentation from:
http://x-colony.com/PoCo/index.php/events-2/crm9-2019/

- XColony Resources for Spatial Cognition
- Example of Activities / School Projects
- Novel Framework for
  - Puzzle Design
  - Problem Posing / Problem Solving
  - Discovery
- PoCo Demo Session
- Educational Programs
- Research Results
- Conclusions & Open Questions

XColony Modules

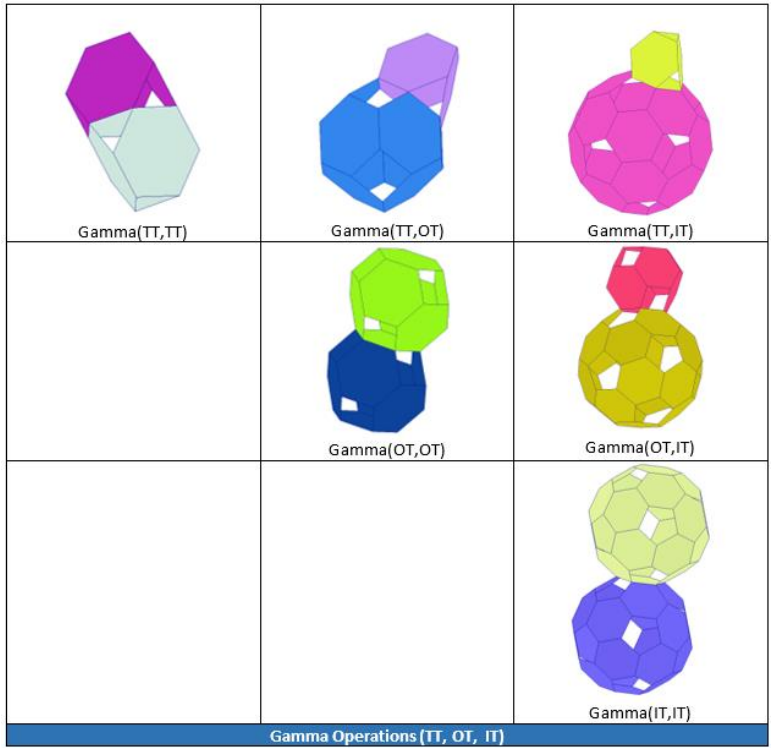


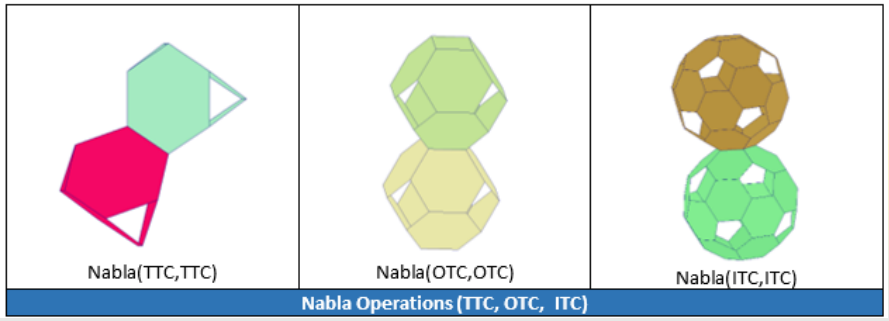
Platonic Solids: **(T)**etrahedron, **(C)**ube, **(O)**ctahedron, **(D)**odecahedron, **(I)**cosahedron



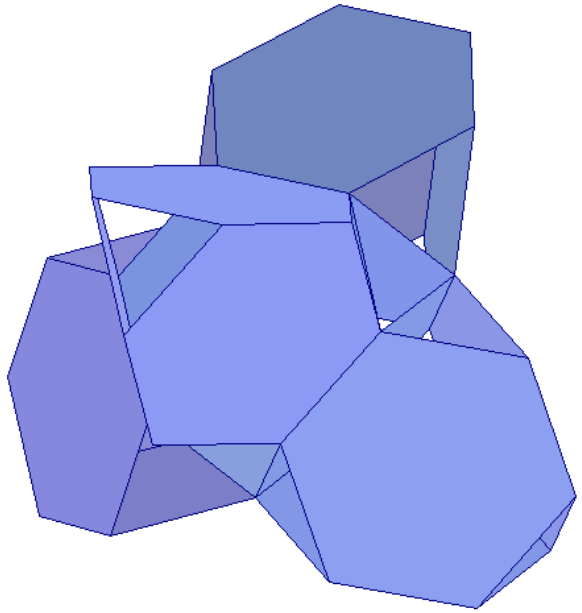
Archimedean Solids (Truncated Platonics): **(TT)**etrahedron, **(OT)**ctahedron, **(IT)**cosahedron
TTC, OTC and ITC – with cofactes

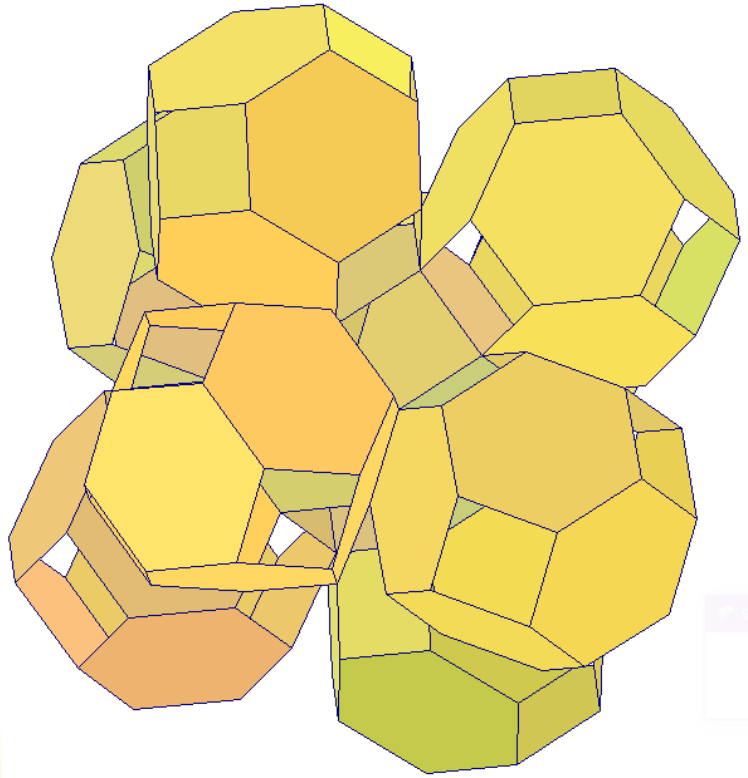
XColony Operations (Delta, Gamma, Nabla)

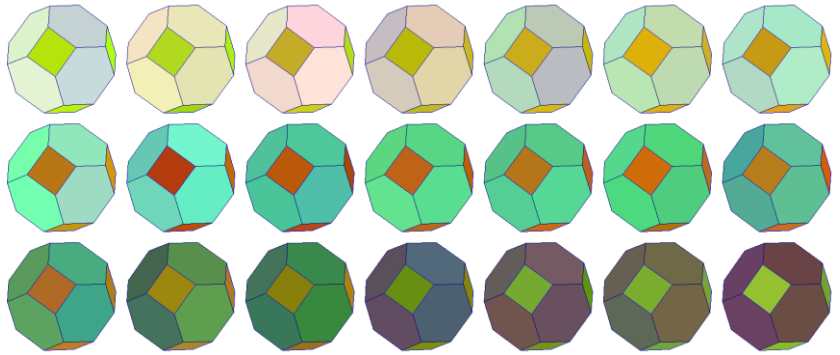


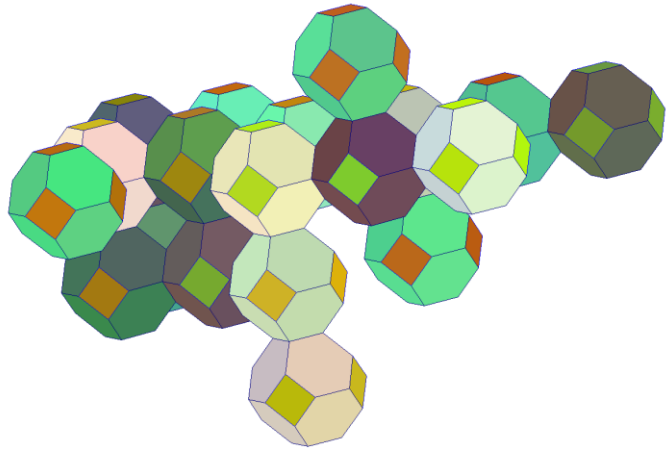


XColony Expressions



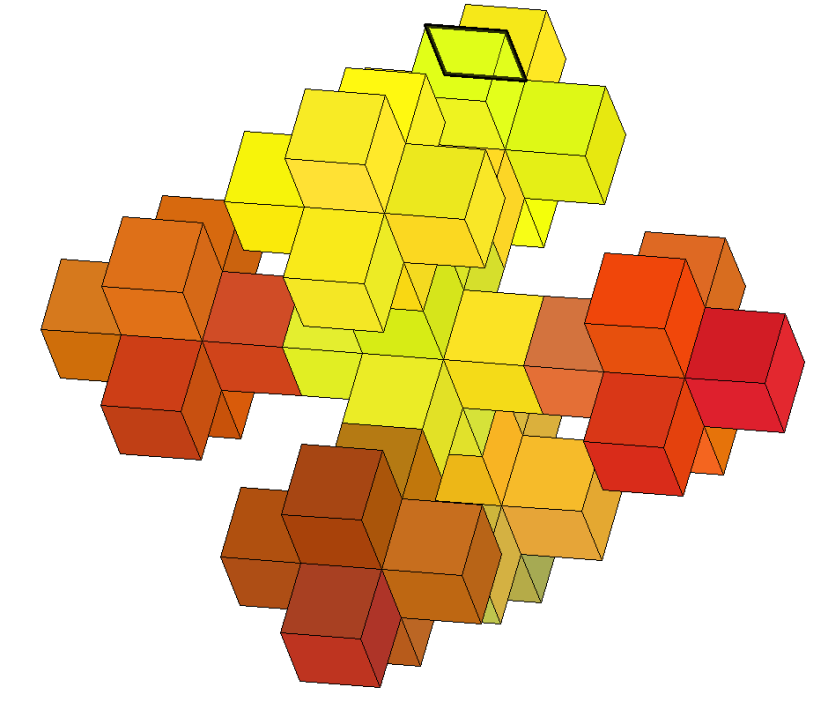




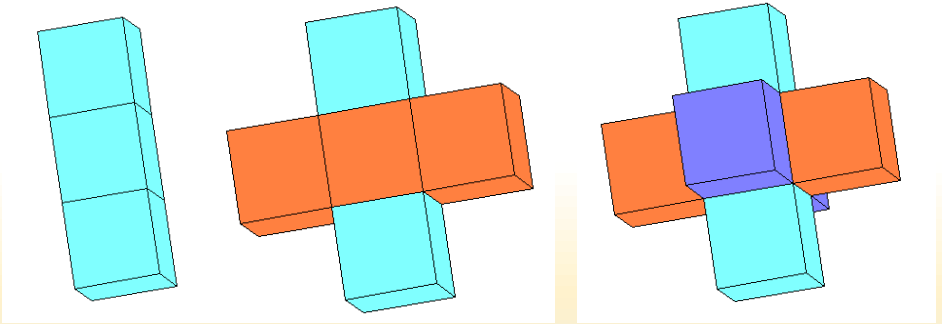


Activities / School Projects (A)

Build the following structure using the smallest number of operations.

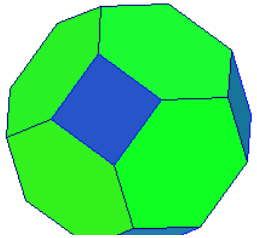


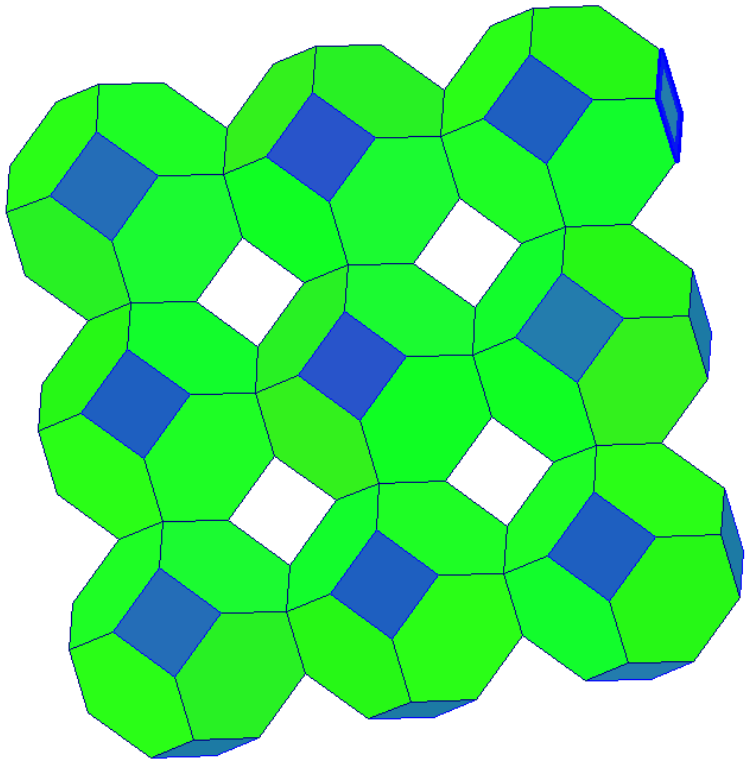
Hint: 4 (2+1+1) Delta operations (carefully choose the binding face)

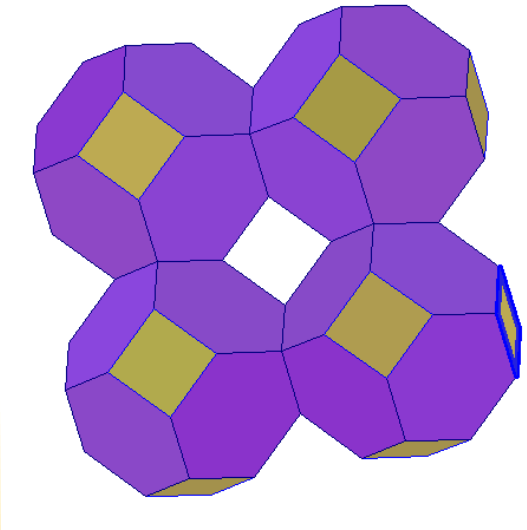


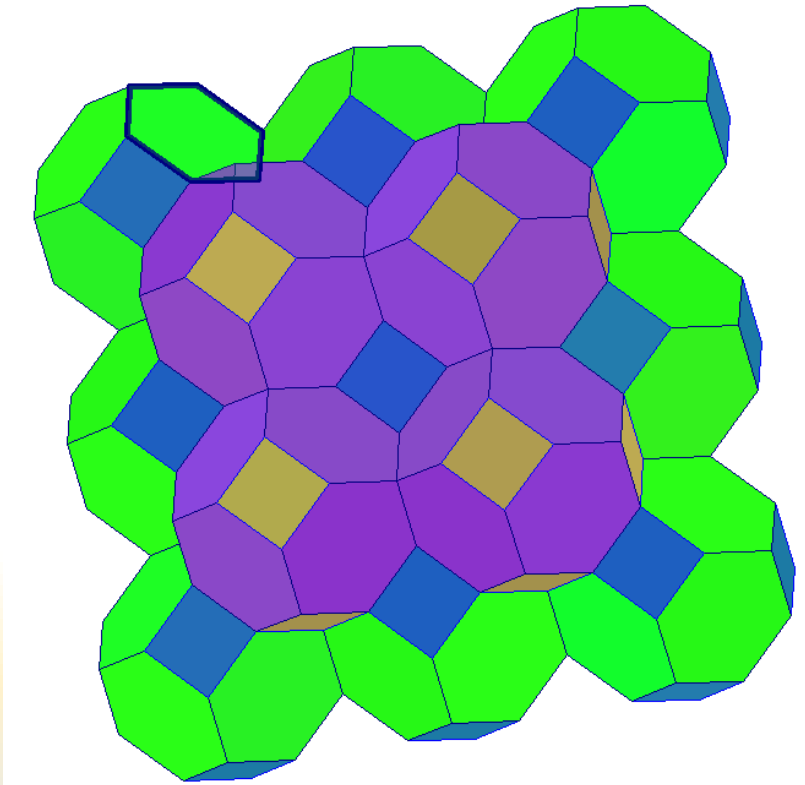
Activities / School Projects (B)

Show that there is a space tessellation with OTC modules.



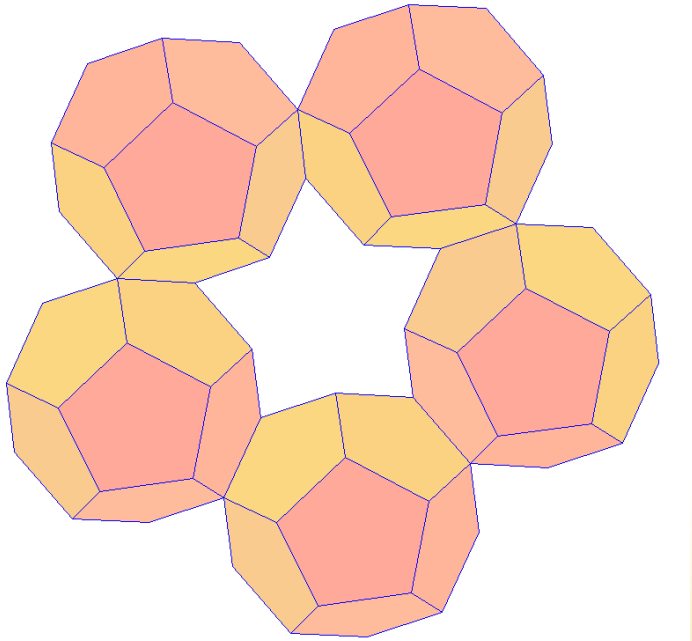






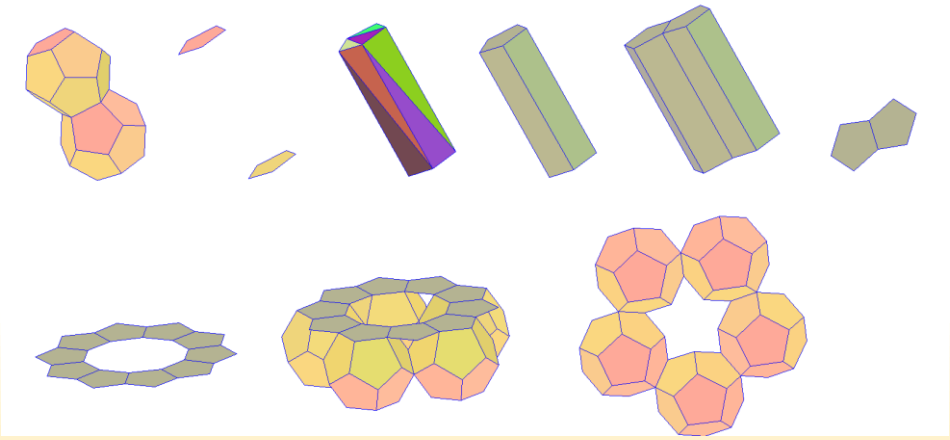
Puzzle Design

Arrange 5 regular dodecahedron on a table such that any 2 neighbors share one full edge, as in the picture below (view from above). Use PoCo to illustrate this.



Solution

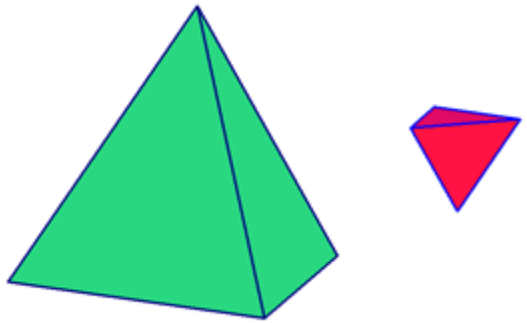
1. Build a pentagonal prism
2. Construct a ring frame of pentagons
3. Attach dodecahedron on any other ones of the pentagons on the frame
4. Delete the pentagons that are not needed.



Problem Solving

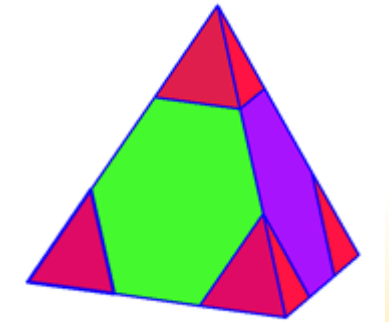
Show that the dual of a regular tetrahedron has the side length equal to 1/3 of the one of the original. Use PoCo framework to prove this.

Show that the corollary: “The dihedral angles of a regular tetrahedron and that of a regular octahedron are supplementary” is true.



Solution

1. Build the truncation of the tetrahedron (1/3 cutting corners)
2. Construct the convex hull and simplify the object
3. Attach 4 copies of the dual to reconstruct the original

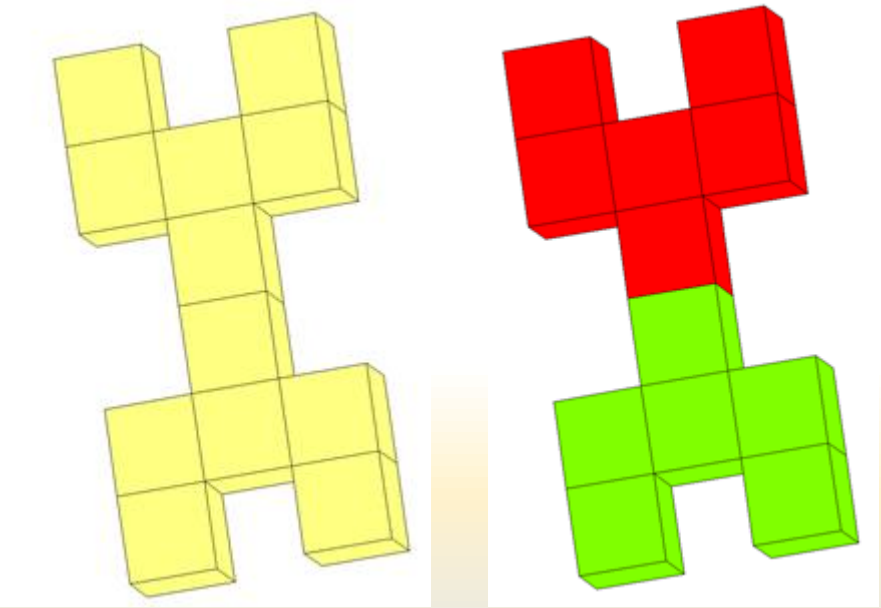


Problem Posing

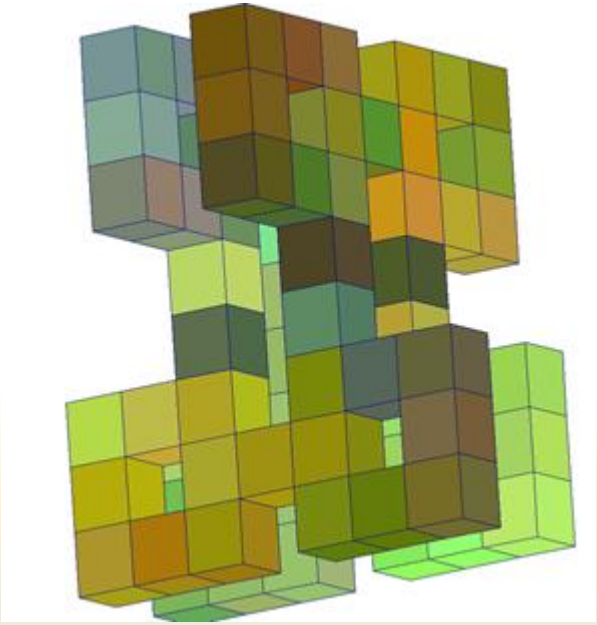
Maximal Module Decomposition

A complex module was obtained by gluing together several cubes. You must identify a maximal module that generates a perfect decomposition of the original object.

Example



New Problem. Find the maximal decomposition module of the ensemble of cubes shown below



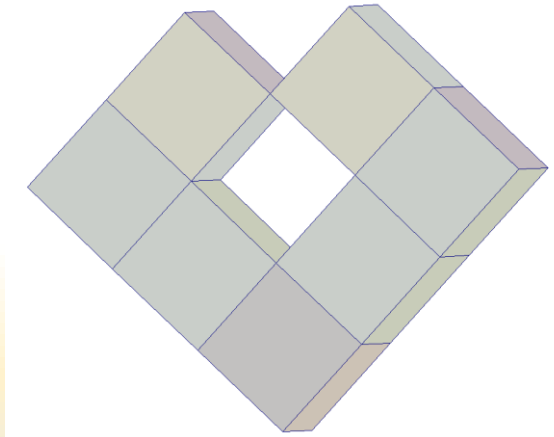
Discovery

Lattice Proximity. Create a lattice of 8 cubes: Start with the cube at the bottom, add 2 cubes in two adjacent directions, change direction and add one more, as shown in the Example below.

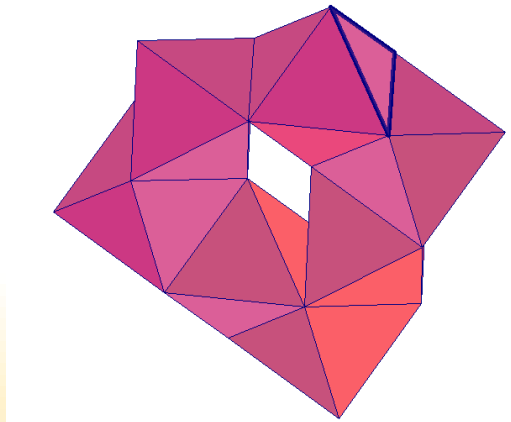
Note that the last two added cubes share a full edge. Also, it is possible to place the Maximal element on top to create a ring structure.

Research Question. For what other modules is this property also valid?

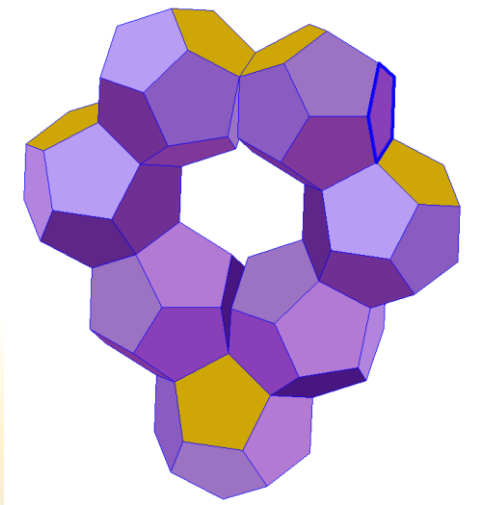
Example



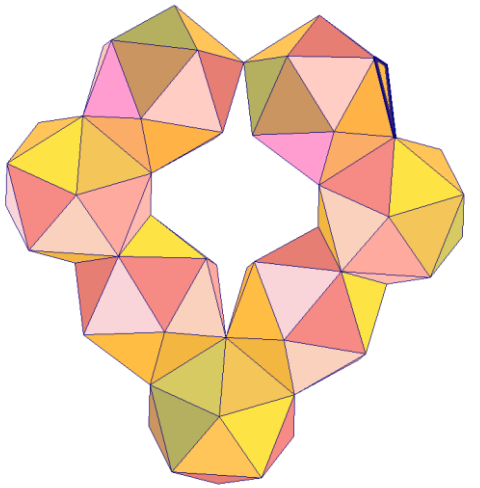
C – True



T – True

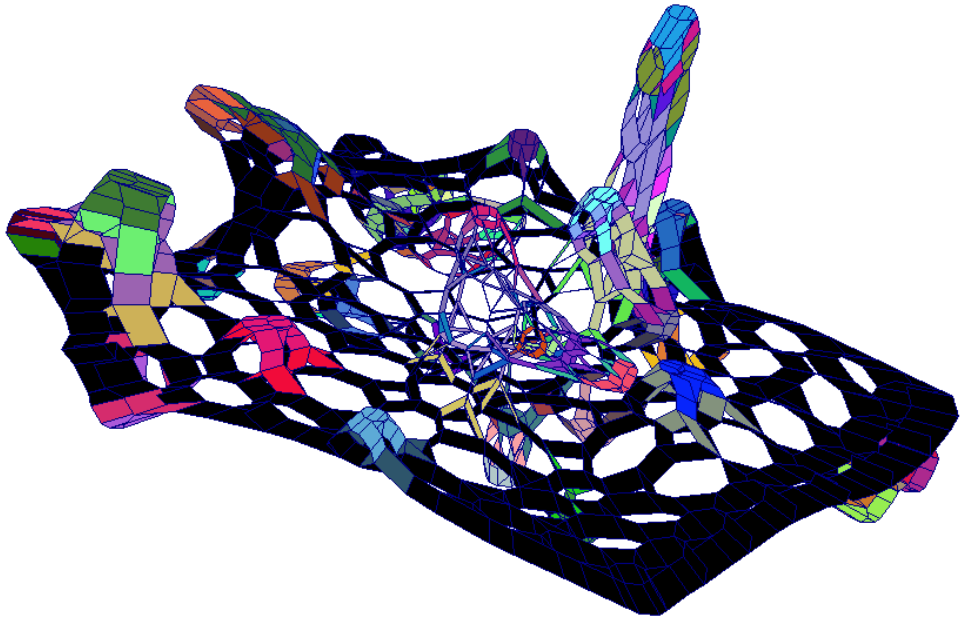


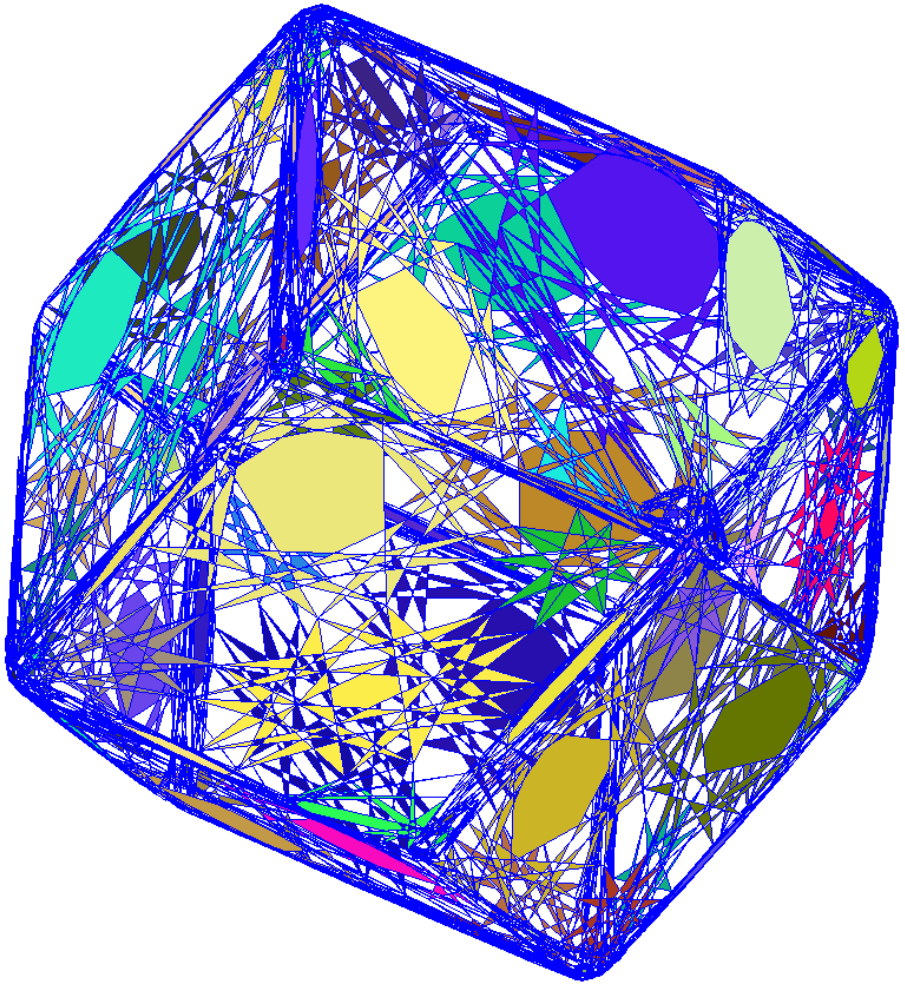
D – True



I – Partially True

PoCo Demo Session





Educational Programs

Spatial Intelligence is difficult to be trained. The collection of currently available tools for teaching spatial intelligence is limited. The tools for evaluation are rudimentary, mostly oriented to 2D questions.

Educational programs based on Polyhedral Computing showed significant improvements in:

- 26% for spatial reasoning
- 15% for solution finding
- 7% for comprehension

The increase in math scores in the next year of the students (now sixth graders) was 12% in standard evaluation tests.

Conclusions & Open Questions

- Development of spatial education programs
- Content creation for spatial learning
- Design of tests for measuring Spatial IQ
- Inclusion of spatial educational modules in standard curriculum
- Investigate within longitudinal studies the long term impact of spatial education for academic performance and career development